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PBIB DESIGNS FOR TWO SETS OF INBRED LINES

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SUMMARY

This paper deals with analysis and construction of partially balanced incomplete block (PBIB) designs with seven associate classes which can be used for confounded diallel experiments involving $v = 2pq$ crosses between two sets of inbred lines to obtain different amount of information on all the seven types of comparisons i.e. comparisons of general combining ability (g.c.a.) effects of the two sets, specific combining ability (s.c.a.) effects, residual reciprocal effects and F_1-crosses v/s reciprocal F_1-crosses.

Keywords : Diallel experiments; Fixed effect model; Mixed model; Orthogonal contrast.

Introduction

The concept of diallel crossing has been immensely useful in analysing genetical potential of a set of lines and in evaluating the best combination of crossing material in breeding programmes. Griffing [7] presented analyses for four diallel crossing schemes depending upon the inclusion of reciprocal crosses and/or parental lines using randomised complete block design (RBD). But for large number of inbred lines, the use of RBD may not be possible. So Kempthorne and Curnow [8] and Arya [6] gave circulant plans for partial diallel crosses. Raghavarao and Aggarwal [13]; Aggarwal [1], [2], [3] and [4]; Arya and Narain [5] and Narain and Arya [10] used PBIB designs for diallel experiments.

Robinson [14] offered analysis for crosses between two sets and within one set. Lin [9] presented RBD analysis for crosses between two sets of lines, coming from two different geographical/genetical back-grounds, including parents. We are introducing PBIB designs, following a new association scheme, which are shown to be useful in analysing the combining abilities of two sets of lines. The often demand for different amount of information on different types of comparisons necessitated the introduction of the following 7-class association scheme.

Definition 1. Let $v = 2pq$ (for $p > 1$ and $q > 1$) treatments be denoted ij and ji ($i = 1, 2, \ldots, p$; $j = p + 1, p + 2, \ldots, p + q$) and the treatment ji be called the reciprocal of the treatment ij. For any treatment ij,

(i) all treatments ij' ($j' \neq j = p + 1, p + 2, \ldots, p + q$) are first assesociates;

(ii) all treatments $j'i$ are second associates;

(iii) all treatments $i'j$ ($i' \neq i = 1, 2, \ldots, p$) are third associates;

(iv) all treatments ji' are fourth associates;

(v) all treatments $i'j'$ are fifth associates;

(vi) all treatments $j'i'$ are sixth associates; and

(vii) the treatment ji is seventh associate.

For any treatment ji, its kth ($k = 1, 2, \ldots, 7$) associates are reciprocals of kth associates of ij.

The parameters of this association scheme will be

$$v = 2pq,\ n_1 = n_2 = q - 1,\ n_3 = n_4 = p - 1,\ n_5 = n_6 = (p - 1)(q - 1),\ n_7 = 1;$$

$$p^1_{27} = p^3_{47} = p^5_{13} = p^5_{24} = p^5_{67} = p^6_{14} = p^6_{23} = 1;$$

$$p^1_{11} = p^1_{22} = p^5_{15} = p^5_{26} = p^6_{16} = q - 2;$$

$$p^3_{33} = p^3_{44} = p^5_{35} = p^6_{36} = p^6_{45} = p - 2;$$

$$p^5_{55} = p^5_{66} = (q - 2)(p - 2);\ \text{and}$$

any other p^i_{jk} ($i, j, k = 1, 2, \ldots, 7$) follows from either

$$p^i_{jk} = p^i_{kj} \text{ or } n_i p^i_{jk} = n_j p^j_{ik}$$

otherwise it is zero.

Example : Let $p = 3$ and $q = 4$

For a treatment 14 (say) its seven associate classes are :

Associate class	*Treatments*					
First	15	16	17			
Second	51	61	71			
Third	24	34				
Fourth	42	43				
Fifth	25	26	27	35	36	37
Sixth	52	62	72	53	63	73
Seventh		41				

Definition 2. The association scheme given in definition 1 will be called 'Extended Right Angular' (ERA) association scheme. The PBIB design following the ERA association scheme will be called an 'Extended Right Angular (ERA) Design'.

Let N be the incidence matrix of an ERA design with the parameters $v = 2pq, b, r\ k, \lambda_i\ (i = 1, 2, \ldots, 7)$. The eigenvalues (θ_i) except $\theta_0 = rk$ of NN' with their multiplicities (α_i^*) $(i = 1, 2, \ldots, 7)$ are given in Table 1. The non-zero eigen-values of the C-matrix of the ERA design will be $\phi_i = r - (\theta_i/k)$ with their multiplicities (α_i^*) $(i = 1, 2, \ldots, 7)$. The ERA design will be connected if $\phi_j \neq 0$ or $\theta_j \neq rk$ or $\sum_{i=1}^{7} c_i\lambda_i \neq r(k-1)$ for any θ_j $(j = 1, 2, \ldots, 7)$ where c_i's are coefficient of λ_i's in θ_j. As $\sum_{i=1}^{7} n_i\lambda_i = r(k-1)$, therefore values of λ_i's should be such that $\sum_{i=1}^{7} (n_i - c_i)\lambda_i \neq 0$ for any θ_j $(j = 1, 2, \ldots, 7)$.

For definitions of various breeding terms used, refer to Aggarwal [2] and for the definitions of various statistical terms and notations used refer to Raghavarao [12].

2. Confounded Diallel* Experiments

Let two sets of inbred lines represent two different geographical/genetical back-grounds. Let the first set contain p inbred lines and the second set contain q inbred lines. Let $2pq$ crosses, involving F_1-crosses and reciprocal F_1-crosses between the two sets of inbred lines, be represented by $v = 2pq$ treatments of a connected ERA design with the parameters v, b, r, k, λ_i $(i = 1, 2, \ldots, 7)$. Let the yield (Y_{ijk}) of the ijth cross allotted to a plot in the kth block be given by

$$Y_{ijk} = m + g_i + g_j + s_{ij} + m_i + n_{ij} + \beta_k + e_{ijk};\ k = 1, 2, \ldots, b \quad (1)$$

*Confounded diallel here means that the partia set of crosses $(2pq)$ from diallel experiment with $(p + q)$ inbred lines is experimented with the concept of confounding explained by Raghavarao [9, p. 245].

where m is the general mean; g_i is the g.c.a. effect of the ith line and so is g_j, s_{ij} is the s.c.a. effect of the ijth cross, m_i is the maternal effect of ith line, n_{ij} is the residual reciprocal effect due to ijth cross and β_k is the effect of kth block. Let (1) be fixed effects model (Scheffe' 15, p. 6) with the restrictions

$$\sum_{i=1}^{p} g_i = \sum_{j=p+1}^{p+q} g_j = 0,\ s_{ij} = s_{ji},\ \sum_{i=1}^{p} s_{ij} = \sum_{j=p+1}^{p+q} s_{ij} = 0,$$

$$m_i + n_{ij} = -(m_j + n_{ji}),\ \sum_{j=p+1}^{p+q} n_{ij} = \sum_{i=1}^{p} n_{ji} = 0 \text{ and } \sum_{k=1}^{b} \beta_k = 0.$$

e_{ijk}'s are assumed to be independently and normally distributed with means zero and variance σ^2.

Considering the sum $g_i + g_j + s_{ij} + m_i + n_{ij}$ in (1) as the split of the true effect (α_{ij}) of the ijth cross, we can obtain the reduced normal equations

$$\underline{C}\ \hat{\underline{\alpha}} = \underline{Q} \tag{2}$$

The orthonormal eigen-vectors $\underline{x}_{ik}$'s, $\underline{y}_{ik}$'s, $\underline{w}_{nk}$'s, $\underline{z}_{nk}$'s and $\underline{x}$ corresponding to non-zero eigen-values (ϕ_i) of the C-matrix of the ERA design represent various types of orthonormalized contrasts given in Table 1. These orthonormal eigen-vectors and eigen-values provide a solution of the reduced normal equations (Raghavarao [11]) which helps in computing the ANOVA TABLE (Table 2). For testing the significance of any effect its mean square (M.S.) is tested against the error M.S.

From the least square estimates ($\hat{\alpha}_{ij}$) of the true effects of the crosses, the least square estimates of various parameters, their variances and variances of their elementary contrasts can be worked out and are given below :

$$\hat{g}_i = (Q_{i.} + Q_{.i})/2q\phi_1;$$

$$\hat{g}_j = (Q_{j.} + Q_{.j})/2p\phi_3;$$

$$\hat{m}_i = (Q_{i.} - Q_{.i})/2q\phi_2 + (1/\phi_7 - 1/\phi_2)\,\Delta_1^Q/pq;$$

$$\hat{m}_j = (Q_{j.} - Q_{.j})/2p\phi_4 + (1/\phi_4 - 1/\phi_7)\,\Delta_1^Q/pq;$$

$$\hat{s}_{ij} = [Q_{ij} + Q_{ji} - (Q_{i.} + Q_{.i})/q - (Q_{j.} + Q_{.j})/p]/2\phi_5;$$

$$\hat{n}_{ij} = (Q_{ij} - Q_{ji})/2\phi_6 - (Q_{i.} - Q_{.i}/2q\phi_6 + (1/\phi_6 - 1/\phi_4)\,(Q_{j.} - Q_{.j})/2p + (1/\phi_6 - 1/\phi_4)\,\Delta_1^Q/pq$$

$$V(\hat{g}_i) = (p-1)\,\sigma^2/2pq\phi_1;$$

$$v(\hat{g}_j) = (q-1)\,\sigma^2/2pq\phi_3;$$
$$v(\hat{m}_i) = [(p-1)/\phi_2 + 1/\phi_7]\,\sigma^2/2pq;$$
$$v(\hat{m}_j) = [(q-1)/\phi_4 + 1/\phi_7]\,\sigma^2/2pq;$$
$$v(\hat{s}_{ij}) = (p-1)\,(q-1)\,\sigma^2/2pq\phi_5;$$
$$v(\hat{n}_{ij}) = [(p-1)/\phi_6 + 1/\phi_4]\,(q-1)\,\sigma^2/2pq;$$
$$v(\hat{g}_i - \hat{g}_{i'}) = \sigma^2/q\phi_1;$$
$$v(\hat{g}_j - \hat{g}_{j'}) = \sigma^2/p\phi_3;$$
$$v(\hat{m}_i - \hat{m}_{i'}) = \sigma^2/q\phi_2;$$
$$v(\hat{m}_j - \hat{m}_{j'}) = \sigma^2/p\phi_4;$$
$$v(\hat{s}_{ij} - \hat{s}_{ij'}) = (p-1)\sigma^2/p\phi_5;$$
$$v(\hat{s}_{ij} - \hat{s}_{i'j}) = (q-1)\,\sigma^2/q\phi_5;$$
$$v(\hat{s}_{ij} - \hat{s}_{i'j'}) = (pq-p-q)\,\sigma^2/pq\phi_5;$$
$$v(\hat{n}_{ij} - \hat{n}_{ij'}) = [(p-1)/\phi_6 + 1/\phi_4]\,\sigma^2/p;$$
$$v(\hat{n}_{ij} - \hat{n}_{i'j}) = (q-1)\,\sigma^2/q\phi_6; \text{ and}$$
$$v(\hat{n}_{ij} - \hat{n}_{i'j'}) = [(pq-p-q)\phi_6 + q/\phi_4]\sigma^2/pq,\ i \neq i' = 1, 2, \ldots, p;$$
$$j \neq j' = p+1, \ldots, p+q.$$

3. Mixed Model

Let m and β_k's in equation (1) be fixed effects and g_i's, g_j's, s_{ij}'s, m_i's, n_{ij}'s, m_j's, n_{ji}'s and e_{ijk}'s be independently and normally distributed with mean zero, variances $\sigma^2_{g_1}$, $\sigma^2_{g_2}$, σ^2, $\sigma^2_{m_1}$, σ^2_n, $\sigma^2_{m_2}$, σ^2_n and σ^2, respectively and pairwise uncorrelated. Under these rerstrictions, the expectations of various mean squares are given in the Table 2. The usual test procedure is followed for testing the significance and estimation of the variance components $\sigma^2_{g_1}$, $\sigma^2_{g_2}$, σ^2_s, $\sigma^2_{m_1}$, $\sigma^2_{m_2}$ and σ^2_n.

4. Methods of Constructing ERA Designs

THEOREM 1. *If $q = p$ is a prime or a prime power then a series of ERA designs with the parameters*

$$v = 2p^2, b = p\,(p-1), r = (p-1), k = 2p, \lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0, \tag{3}$$

$\lambda_5 = \lambda_6 = 1$, $\lambda_7 = (p-1)$, *can always be constructed.*

TABLE 1—SETS OF ORTHONORMALIZED CONTRASTS

Sr. No.	*roots of NN′*	*multiplicities*	*contrasts*	*effects represented*
1.	$\theta_1 = r + \lambda_7 + (q-1)(\lambda_1 + \lambda_2 - \lambda_5 - y_6) - (\lambda_3 + \lambda_4)$	$\alpha_1^* = (p-1)$	$\underline{\alpha}'\underline{x}_{1k} = \left[\sum_{i=1}^{k} (\alpha_{i.} + \alpha_{.i}) - k\,(\alpha_{(k+1).} + \alpha_{.(k+1)})\right] \div [2qk\,(k+1)]^{1/2}, k = 1, 2, \ldots, p-1;$	g.c.a. effects of first set
2.	$\theta_2 = r - \lambda_7 + (q-1)(\lambda_1 - \lambda_2 - \lambda_5) + \lambda_6) - (\lambda_3 - \lambda_4)$	$\alpha_2^* = (p-1)$	$\underline{\alpha}'\underline{y}_{1k} = \left[\sum_{i=1}^{k} (\alpha_{i.} - \alpha_{.i}) - k\,(\alpha_{(k+1).} - \alpha_{.(k+1)})\right] \div [2qk\,(k+1)]^{1/2}, k = 1, \ldots, p-1;$	maternal effects of the first set
3.	$\theta_3 = r + \lambda_7 - (\lambda_1 + \lambda_2) + (p-1)(\lambda_3 + \lambda_4 - \lambda_5 - \lambda_6)$	$\alpha_3^* = (q-1)$	$\underline{\alpha}'\underline{x}_{2k} = \left[\sum_{j=p+1}^{p+k} (\alpha_{j.} + \alpha_{.j}) - k(\alpha_{(p+k+1).} + \alpha_{.(p+k+1)})\right] \div [2pk\,(k+1)]^{1/2}, k = 1, 2, \ldots, q-1;$	g.c.a effects of the second set
4.	$\theta_4 = r - \lambda_7 - (\lambda_1 - \lambda_2) + (p-1)(\lambda_3 - \lambda_4 - \lambda_5 + \lambda_6)$	$\alpha_4^* = (q-1)$	$\underline{\alpha}'\underline{y}_{2k} = \left[\sum_{j=p+1}^{p+k} (\alpha_{j.} - \alpha_{.j}) - k(\alpha_{(p+k+1).} - \alpha_{.(p+k+1)})\right] \div [2pk\,(k+1)]^{1/2}, k = 1, 2, \ldots, q-1;$	maternal effects of the second set
5.	$\theta_5 = r + \lambda_7 - (\lambda_1 + \lambda_2) - (\lambda_3 + \lambda_4) + (\lambda_5 + \lambda_6)$	$\alpha_5^* = (p-1)(q-1)$	$\underline{\alpha}'\underline{z}_{nk} = \left[\sum_{i=1}^{n} \left\{\sum_{j=p+1}^{p+k} (\alpha_{ij} + \alpha_{ji}) - k\,(\alpha_{i(p+k+1)} + \alpha_{(p+k+1)i})\right\} - n\left\{\sum_{j=p+1}^{p+k} (\alpha_{(n+1)j} + \alpha_{j(n+1)}) - k(\alpha_{(n+1)(p+k+1)} + \alpha_{(p+k+1)(n+1)})\right\}\right] \div [2n\,(n+1)\,k(k+1)]^{1/2}, n = 1, 2, \ldots, p-1;\ k = 1, 2, \ldots, q-1;$	s. c. a. effects

6.	$\theta_6 = r - \lambda_7 - (\lambda_1 - \lambda_2) - (\lambda_3 - \lambda_4) + (\lambda_5 - \lambda_6)$	$\alpha_6^* = (p-1)(q-1)$	$\underline{\alpha}'\, \underline{w}_{nk} = \left[\sum_{i=1}^{n} \left\{ \sum_{j=p+1}^{p+k} (\alpha_{ij} - \alpha_{ji}) - k(\alpha_{i(p+k+1)} - \alpha_{(p+k+1)i}) \right\} n \left\{ \sum_{j=p+1}^{p+k} (\alpha_{(n+1)j} - \alpha_{j(n+1)}) - k(\alpha_{(n+1)\,(p+k+1)} - \alpha_{(p+k+1)\,(n+1)}) \right\} \right] \div [2n\,(n+1)\,k(k+1)]^{1/2}$; $n = 1, 2, \ldots, p-1$; $k = 1, 2, \ldots, q-1$;	residual reciprocal effects
7.	$\theta_7 = r - \lambda_7 + (q-1)(\lambda_1 - \lambda_2) + (p-1)(\lambda_3 - \lambda_4) + (p-1)(q-1)(\lambda_5 - \lambda_6)$	$\alpha_7^* = 1$	$\underline{\alpha}'\, \underline{x} = [\Delta_1^{\alpha} - \Delta_2^{\alpha}] + (2pq)^{1/2}$	F_1 crosses v/s reciprocal F_1- crosses

where $\underline{\alpha} = [\alpha_{1(p+1)}\, \alpha_{1(p+2)}, \ldots, \alpha_{1(p+q)}, \alpha_{2(p+1)}\, \alpha_{2(p+2)}, \ldots, \alpha_{(p+q)1}, \alpha_{(p+q)2}, \alpha_{(p+q)3}, \alpha_{(p+q)4}, \ldots, \alpha_{(p+q)p}]'$

$$\alpha_{i.} = \sum_{j=p+1}^{p+q} \alpha_{ij},\ \alpha_{.i} = \sum_{j=p+1}^{p+q} \alpha_{ji},\ \Delta_1^{\alpha} = \sum_{i=1}^{p} \alpha_{i.} \text{ and } \Delta_2^{\alpha} = \sum_{i=1}^{p} \alpha_{.i}$$

TABLE 2—ANOVA TABLE

Source	*d.f.*	*S. S.*	*E(M.S.) Mixed-effects Model*
Blocks ignoring treatments	$(b-1)$	$\sum_{i=1}^{b} (y^2 \ldots 1)/k - y^2 \ldots /rv$	—
g.c.a. effects of the first set	$(p-1)$	$\left[\sum_{i=1}^{p} (Q_{i.} + Q_{.i})^2\right] \Big/ 2q\phi_1$	$\sigma^2 + 2\phi_1 [\sigma_s^2 + q\sigma_{g_1}^2]$
maternal effects of the first set	$(p-1)$	$\left[\sum_{i=1}^{p} (Q_{i.} - Q_{.i})^2 - (2\Delta_1^Q)^2/p\right] \Big/ 2q\phi_2$	$\sigma^2 + 2\phi_2 [\sigma_n^2 + q\sigma_{m_1}^2]$
g.c a. effects of the second set	$(q-1)$	$\left[\sum_{j=p+1}^{p+q} (Q_{j.} + Q_{.j})^2\right] \Big/ 2p\phi_3$	$\sigma^2 + 2\phi_3 [\sigma_s^2 + p\sigma_{g_2}^2]$
maternal effects of the second set	$(q-1)$	$\left[\sum_{j=p+1}^{p+q} (Q_{j.} - Q_{.j})^2 - (2\Delta_1^Q)^2/q\right] \Big/ 2p\phi_4$	$\sigma^2 + 2\phi_4 [\sigma_n^2 + p\sigma_{m_2}^2]$
s.c.a. effects	$(p-1)(q-1)$	$\left[\sum_{i=1}^{p} \sum_{j=p+1}^{p+q} (Q_{ij} + Q_{ji})^2 - \sum_{i=1}^{p} (Q_{i.} + Q_{.i})^2/q - \sum_{j=p+1}^{p+q} (Q_{j.} + Q_{.j})^2/p\right] \Big/ 2\phi_5$	$\sigma^2 + 2\phi_5 \sigma_s^2$

residual reciprocal effects	$(p-1)(q-1)$	$\left[\sum_{i=1}^{p}\sum_{j=p+1}^{p+q}(Q_{ij}-Q_{ji})^2-\sum_{i=1}^{p}(Q_{i.}-Q_{.i})^2/q - \sum_{j=p+1}^{p+q}(Q_{j.}-Q_{.j})^2/p+(2\Delta_1^Q)^2/pq\right]\Big/2\phi_6$	$\sigma^2+2\phi_6\,\sigma_n^2$
F_1-crosses v/s reciprocal F_1-crosses	1	$2(\Delta_1^Q)^2/qp\phi_7$	$\sigma^2+2\phi_7\,[\sigma_n^2+q\sigma_{m_1}^2]$
Error	$(vr-b-v+1)$	By subtraction	σ^2
Total $vr-1$		$\sum_i\sum_j\sum_k Y_{ijk}^2-Y^2_{...}/rv$	

where $Q_{i.}$, $Q_{.i}$, Δ_1^Q and Δ_2^Q can be analogously defined as in Table 1.

Proof. Put p^2 crosses in a $p \times p$ square array such that ith row $(i = 1, 2, \ldots, p)$ contains crosses $i \times j$ $(j = p + 1, \ldots, 2p)$ and the jth column $(j = 1, 2, \ldots, p)$ contains crosses $i \times (p + j)$ $(i = 1, 2, \ldots, p)$. Superimpose each of the $(p - 1)$ MOLS on this square array. Putting all the crosses which occur with the same letter of a latin square in a block, we can get p blocks with each latin square. Thus $(p - 1)$ MOLS give $p(p - 1)$ blocks. Doubling the block size by adding the cross $j \times i$ for each cross $i \times j$ $(i = 1, 2, \ldots, p;\ j = p + 1, \ldots, 2p)$ in these blocks, a series of ERA designs with the parameters given in (3) can be obtained.

THEOREM 2. *A Series of ERA designs with the parameters*

$$v = 2pq,\ b = b^*,\ r = \lambda_1 = \lambda_2 = \lambda_7 = r^*,\ k = 2qk^*,\ \lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = \lambda^* \tag{4}$$

where $v^* = p$, b^*, r^*, k^*, λ^* *are the parameters of a BIB design, can always be constructed.*

Proof. Construct p groups of $2pq$ crosses such that ith $(i = 1, 2, \ldots, p)$ group contains crosses $i \times j, j \times i$ $(j = p + 1, \ldots, p + q)$. Define a one-one correspondence between these p groups and the p treatments of a BIB design with the parameters $v^* = p$, b^*, r^*, k^*, λ^*. Replace each treatment of the BIB design by all the crosses in the corresponding group. Thus we get a series of ERA designs with the parameters given in (4).

For the series of ERA designs given in (3), the relative loss of information (Shah [16]) is only on d.f. pertaining to s.c.a. effects and that is $1/(p - 1)$ for each d.f. For the series of ERA designs given in (4), the relative loss of information is only on d.f. pertaining to g.c.a. effects of the first set and that is $(r^* - \lambda^*)/r^*k^*$ for each d.f.

5. Worked Example

Let $v = 24$ (for $p = 3$ and $q = 4$), $b = 6$, $r = 3$, $k = 12$,

$\lambda_1 = \lambda_2 = \lambda_5 = \lambda_6 = 1$ and $\lambda_3 = \lambda_4 = \lambda_7 = 3$ be the parameters of an ERA design. The yields of the crosses $i \times j, j \times i$ $(i = 1, 2, \ldots, p;\ j = p + 1, p + 2, \ldots, p + q)$ are given within brackets in the following 6 blocks.

$$\begin{bmatrix} 1\times4 & 2\times4 & 3\times4 & 4\times1 & 4\times2 & 4\times3 & 1\times5 & 2\times5 & 3\times5 & 5\times1 & 5\times2 & 5\times3 \\ (11) & (1) & (3) & (5) & (8) & (10) & (9) & (5) & (5) & (8) & (2) & (2) \end{bmatrix}$$

$$\begin{bmatrix} 1\times4 & 2\times4 & 3\times4 & 4\times1 & 4\times2 & 4\times3 & 1\times6 & 2\times6 & 3\times6 & 6\times1 & 6\times2 & 6\times3 \\ (7) & (3) & (6) & (8) & (1) & (5) & (6) & (2) & (3) & (9) & (11) & (9) \end{bmatrix}$$

$$\begin{bmatrix} 1\times4 & 2\times4 & 3\times4 & 4\times1 & 4\times2 & 4\times3 & 1\times7 & 2\times7 & 3\times7 & 7\times1 & 7\times2 & 7\times3 \\ (9) & (2) & (5) & (4) & (12) & (10) & (11) & (7) & (8) & (3) & (2) & (5) \end{bmatrix}$$

$$\begin{bmatrix} 1\times5 & 2\times5 & 3\times5 & 5\times1 & 5\times2 & 5\times3 & 1\times6 & 2\times6 & 3\times6 & 6\times1 & 6\times2 & 6\times3 \\ (8) & (2) & (5) & (1) & (11) & (6) & (10) & (4) & (1) & (3) & (6) & (12) \end{bmatrix}$$

$$\begin{bmatrix} 1\times5 & 2\times5 & 3\times5 & 5\times1 & 5\times2 & 5\times3 & 1\times7 & 2\times7 & 3\times7 & 7\times1 & 7\times2 & 7\times3 \\ (10) & (3) & (6) & (7) & (4) & (2) & (5) & (1) & (11) & (9) & (7) & (12) \end{bmatrix}$$

$$\begin{bmatrix} 1\times6 & 2\times6 & 3\times6 & 6\times1 & 6\times2 & 6\times3 & 1\times7 & 2\times7 & 3\times7 & 7\times1 & 7\times2 & 7\times3 \\ (13) & (5) & (3) & (1) & (15) & (2) & (4) & (6) & (5) & (8) & (9) & (7) \end{bmatrix}$$

The roots ϕ_i of the C-matrix will be $\phi_1 = \phi_2 = \phi_4 = \phi_5 = \phi_6 = \phi_7 = 3$ and $\phi_8 = 2$. The values of Q_{ij}'s are given in Table (3). Assum-

TABLE 3—CALCULATION OF Q_{ij}'s

Cross ($i \times j$)	T_{ij}	$B^{ij}/12$	Q_{ij}	*Cross* ($j \times i$)	T_{ji}	$B^{ji}/12$	Q_{ji}
1 × 4	27	18.08	8.92	4 × 1	17	18.08	—1.08
1 × 5	27	17.92	9.08	4 × 2	21	18.08	2.92
1 × 6	21	18.08	2.92	4 × 3	25	18.08	6.92
1 × 7	20	19.42	0.58	5 × 1	16	17.92	—1.92
2 × 4	6	18.08	—12.08	5 × 2	17	17.92	—0.92
2 × 5	10	17.92	—7.92	5 × 3	10	17.92	—7.92
2 × 6	11	18.08	—7.08	6 × 1	13	18.08	—5.08
2 × 7	14	19.42	—5.42	6 × 2	32	18.08	13.92
3 × 4	14	18.08	—4.08	6 × 3	23	18.08	4.92
3 × 5	16	17.92	—1.92	7 × 1	20	19.42	0.58
3 × 6	15	18.08	—3.08	7 × 2	18	19.42	—1.42
3 × 7	24	19.42	4.58	7 × 3	24	19.42	4.58

ing various effects to be fixed, the anova table for testing various effects is given in Table (4). From the anova table, it can be observed that only the maternal effects of set 1 are significant at 5 per cent level. The least square estimates of m_1, m_2 and m_3 are 1.21, —1.96 and —0.54, respectively and estimate of SE $(\hat{m}_i - \hat{m}_{i'})$ is 1.04, $i \neq i' = 1, 2, 3$. where T_{ij} is the total of all observations from plots to which $i \times j$ is allotted, B^{ij} is the total of all blocks in which $i \times j$ occurs and the adjusted cross total $Q_{ij} = T_{ij} - B^{ij}/k$.

TABLE 4—ANOVA

Source		*d.f.*	*S. S.*	*M. S.*	*F-ratio*
blocks ignoring treatment		5	8.79	—	—
g c.a. effects	Set 1	2	22.33	11.17	0.86
maternal effects		2	120.78	60.39	4.63*
g.c.a. effects	Set 2	3	15.80	5.27	0.40
maternal effects		3	31.82	10.61	0.81
s. c. a. effects		6	61.57	10.26	0.79
residual reciprocal effects		6	40.22	6.70	0.51
F_1-crosses v/s reciprocal F_1-crosses		1	13.35	13.35	1.02
Error		43	561.21	13.05	—
Total		71	875.87	—	—

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